Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2018

Question/Answer booklet

MATHEMATICS SPECIALIST

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Section One: Calculator-free

Student number: In figures

In words

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	9	9	50	51	35
Section Two: Calculator-assumed	11	11	100	91	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2018*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (51 Marks)**

This section has **nine** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1 **(5 marks)**

Functions f, g are defined such that:

$$f(x) = \sqrt{x - 3}$$

$$g(x) = \frac{x}{x - 2}$$

(a) Determine $g \circ f(x)$. (1 mark)

(b) Determine the domain for $g \circ f(x)$. (2 marks)

(c) Given that $f^{-1}(x) = x^2 + 3$, is it true that $f^{-1}(-1) = 4$? (2 marks)

Explain.

Question 2 **(6 marks)**

(a) Solve the following system of equations: (3 marks)

$$4x - y - 2z = 5$$
$$2x + y - z = 4$$
$$x - y - z = 3$$

See next page

Consider another set of equations where k is a constant.

$$2x - y - z = 0$$
$$x - 2y - z = 2$$
$$x - 2y + kz = 6$$

It can be shown that this system of equations can be reduced to the following:

$$x = \frac{-2\,(k-1)}{3\,(k+1)}$$
$$y = \frac{-4\,(k+2)}{3\,(k+1)}$$
$$z = \frac{4}{k+1}$$

(b) Explain whether this system of equations will have a unique solution for all real values of k. If not, explain the geometric interpretation of this. (3 marks)

Question 3 **(9 marks)**

(a) Let $z = a + bi$ be any complex number. (3 marks)

Obtain an equation relating a, b given that $\text{Re}\left(\frac{z - i}{z}\right) = 0$.

(b) Let $z = r\,cis\theta$ be any complex number. Obtain an expression for:

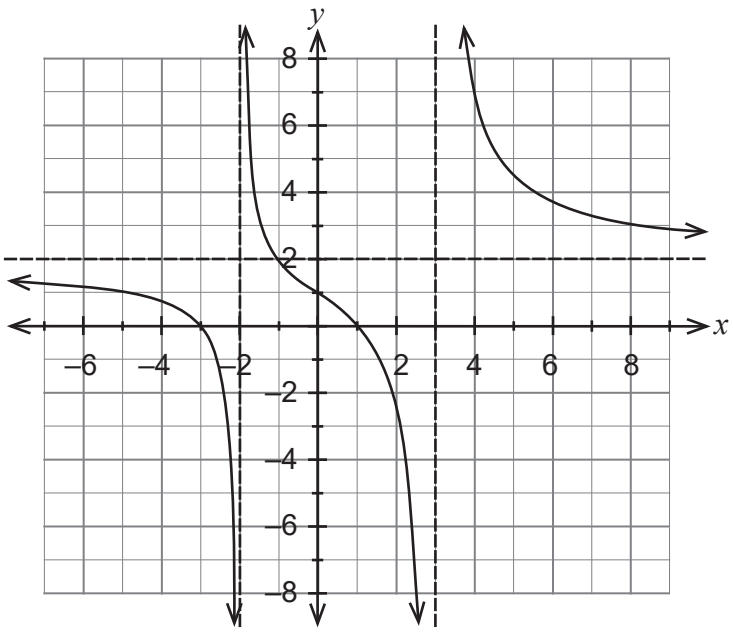
(i) $\frac{2i}{\bar{z}}$ in terms of r, θ. (3 marks)

(ii) $arg\,(z + r)$ in terms of θ. (3 marks)

See next page

Question 4 **(4 marks)**

The graph of $f(x) = \dfrac{k(x-a)(x-b)}{(x-c)(x-d)}$ is shown below.



Determine the value of the constants a, b, c, d and k.

a	b	c	d	k

Explain your choice for the value of k.

Question 5 **(4 marks)**

Using the substitution $u = \cos(2x)$, evaluate exactly the definite integral

$$\int_0^{\frac{\pi}{4}} \cos^{1008}(2x)\sin(2x)\,dx.$$

See next page

Question 6 **(5 marks)**

(a) Given that $\frac{2}{(x+1)(x-1)} = \frac{a}{x-1} + \frac{b}{x+1}$ determine the values for a and b. (2 marks)

(b) Hence determine $\int \frac{1}{x^2-1}\,dx$. (3 marks)

See next page

Question 7 **(6 marks)**

(a) Solve the equation $z^3 + 1 = 0$ giving solutions in polar form $r\,cis\theta$. (3 marks)

It can be shown that $P(z) = z^5 - 2z^4 + 5z^3 + z^2 - 2z + 5$ can be written in the form

$P(z) = (z^3 + 1)Q(z)$.

(b) Determine $Q(z)$. (1 mark)

(c) Hence solve the equation $z^5 - 2z^4 + 5z^3 + z^2 - 2z + 5 = 0$ giving all solutions in Cartesian form $a + bi$. (2 marks)

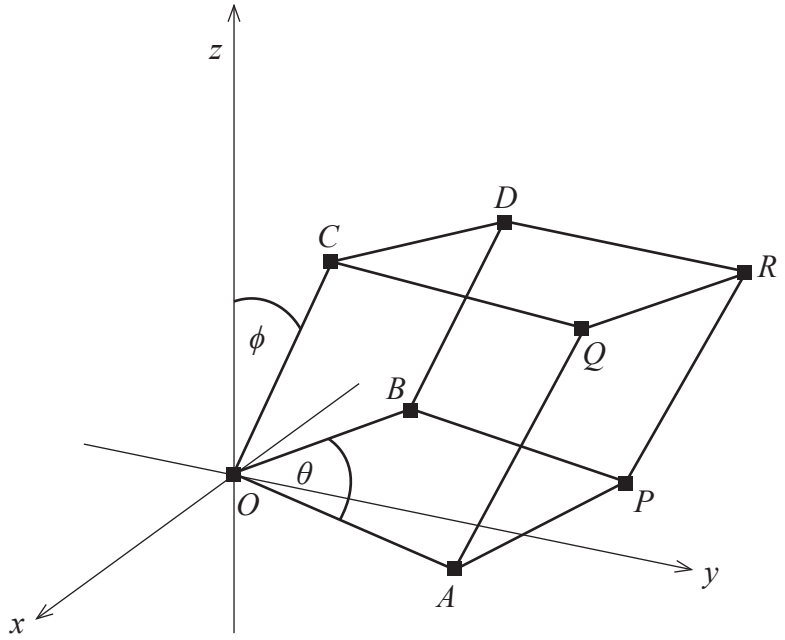
See next page

Question 8 **(5 marks)**

A parallelepiped is a prism where each face is a parallelogram. Let $OAPB$ be the parallelogram formed by the horizontal sides $\underset{\sim}{a} = \overrightarrow{OA}$ and $\underset{\sim}{b} = \overrightarrow{OB}$ where

$\underset{\sim}{a} = \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} -8 \\ 2 \\ 0 \end{pmatrix}$.

The third side that forms the parallelepiped is $\underset{\sim}{c} = \overrightarrow{OC}$ where $\underset{\sim}{c} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$.



Let θ = the size of $\angle AOB$
ϕ = the angle between $\overrightarrow{OC}$ and the positive z axis

(a) Determine $\underset{\sim}{a} \times \underset{\sim}{b}$. (2 marks)

The volume of any prism can be found by considering the formula $Volume = Area\ (Base) \times h$, where h = the perpendicular height of the prism.

It is also true that $|\underset{\sim}{a} \times \underset{\sim}{b}| = |\underset{\sim}{a}||\underset{\sim}{b}| \sin\theta$.

(b) Explain why $\underset{\sim}{c} \cdot (\underset{\sim}{a} \times \underset{\sim}{b})$ will determine the volume of the parallelepiped. (2 marks)

(c) Hence determine the exact volume of the parallelepiped. (1 mark)

Question 9 **(7 marks)**

(a) By using an appropriate trigonometric identity, simplify in terms of u, the expression $x^2 - 2x + 4$ where $x = \sqrt{3}\tan(u) + 1$. (2 marks)

(b) Hence evaluate $\int_1^2 \frac{dx}{\left(x^2 - 2x + 4\right)^{\frac{3}{2}}}$ exactly. (5 marks)

End of section

Supplementary page

Question number: _____

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